

Holographic Hadronization and Thermal Hadron Emission Rate in $\mathcal{N} = 4$ super Yang-Mills Plasma on the Coulomb Branch

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We study $\mathcal{N} = 4$ super Yang-Mills theory on the Coulomb branch (cSYM) in the strong coupling limit by using the AdS/CFT correspondence. The dual geometry is the rotating black 3-brane Type IIB supergravity solution with a single non-zero rotation parameter r_0 which sets a fixed mass scale corresponding to the scalar condensate $\langle \mathcal{O} \rangle \sim r_0^4$ in the coulomb branch. We introduce a new ensemble where T and r_0 are held fixed, and show that r_0 plays a similar role as Λ_{QCD} . We compute the equation of state (EoS) of $\mathcal{N} = 4$ cSYM at finite T , as well as the heavy quark-antiquark potential and the quantized mass spectrums of the scalar and spin-2 glueballs at $T = 0$. By computing the Wilson loop (minimal surface) at $T = 0$, we determine the heavy quark-antiquark potential $V(L)$ to be Cornell potential. At $T \neq 0$, we find two black hole branches: the large black hole and small black hole branches. For the large black hole branch, that has positive specific heat, we find qualitatively similar EoS to that of pure Yang-Mills theory on the lattice. For the small black hole branch, that has negative specific heat and Hawking radiate, we find an EoS where the entropy and energy densities decrease with T . Moreover, we show that the large and small black holes are connected to each other by a second-order phase transition. We conjecture that the small black hole branch should be identified as dual to a thermal gauge theory ($\mathcal{N} = 4$ cSYM) in its hadronizing phase. We argue that our conjecture naturally resolves the small black hole information paradox. We also propose a formula which relates the Hawking radiation rate with the thermal hadron emission rate, and in the hydrodynamic limit reduces to Cooper-Frye formula in the local rest frame.

Introduction. The AdS/CFT correspondence [1–3] has opened a new window to the strongly coupled regime of gauge theories such as $\mathcal{N} = 4$ super Yang-Mills (SYM). Unfortunately, so far, we lack an exact string theory dual to QCD even though there are various works which explored different non-conformal deformations of $\mathcal{N} = 4$ SYM both on the top-down (where both the details of the deformation of $\mathcal{N} = 4$ SYM and its string theory dual are known), and bottom-up approaches (where the details of the deformation of $\mathcal{N} = 4$ SYM and its string theory dual are unknown).

The main goal of both approaches is finding a gravitational background solution that capture some of the essential features of QCD, such as running of the coupling constant, confinement, mass-gap, QCD-like equation of state, etc. Some of the bottom-up approaches include hard-wall model [4, 5], soft-wall models [6–8], approximate black hole duals [9–11], improved holographic QCD [12], while the top-down approaches include $D3/D7$ model [13–15], Witten-Sakai-Sugimoto model [16–18], $\mathcal{N} = 4$ SYM on sphere [16], $\mathcal{N} = 4$ SYM on the Coulomb branch at zero temperature [19–22], and other top-down models [23–25].

In $\mathcal{N} = 4$ SYM on the Coulomb branch at zero temperature, a scale is introduced dynamically through the Higgs mechanism where the scalar particles Φ_i ($i=1\dots 6$) of $\mathcal{N} = 4$ SYM acquire a non-zero vacuum expectation value (VEV) that breaks the conformal symmetry, and the gauge symmetry $SU(N_c)$ to its subgroup $U(1)^{N_c-1}$ without breaking the supersymmetry, and without resulting in a running of the coupling constant [19]. At finite temperature, the mechanism is the same except the fact that supersymmetry will be broken as well.

The string theory dual for $\mathcal{N} = 4$ SYM on the Coulomb branch at zero temperature is well known. Among various Type IIB supergravity background solutions that are dual to the strongly coupled $\mathcal{N} = 4$ SYM on the Coulomb branch at zero temperature [19–22], in this Letter, we will study a Type IIB supergravity background solution that describes non-extremal rotating black 3-branes (with mass parameter m and single rotational parameter r_0) which, in the extremal limit, i.e., $r_0 \gg m^{1/4}$, is dual to $\mathcal{N} = 4$ SYM on the Coulomb branch at zero temperature that arises from N_c D3-branes distributed uniformly in the angular direction, inside a 3-sphere of radius r_0 [20]. In Euclidean space, this rotating black 3-brane background solution has already been shown in [20] to result in confining potential for heavy quarks, and quantized mass spectrum for scalar bulk fluctuations in the extremal limit.

So far the studies of the non-extremal rotating black 3-brane supergravity backgrounds has been limited to the grand canonical ensemble (which is described by fixed temperature T and angular velocity Ω), and canonical ensemble (which is described by fixed temperature T and angular momentum density J). From the field theory perspective these ensembles corresponds to studying $\mathcal{N} = 4$ SYM either at finite T and chemical potential μ (which is the grand canonical ensemble) or at finite T and charge density $\langle J^0 \rangle = \rho$ (canonical ensemble), see [26–36]. The two ensembles have different physics, for example, in planar rotating black 3-branes, Hawking-Page phase transition does not exist in the grand canonical ensemble even though it does exist in the canonical ensemble [30, 35].

In this Letter, we will introduce a new ensemble which

is described by a fixed temperature T and an energy scale Λ which is directly related to the rotation parameter r_0 of the rotating black 3-brane background through $\Lambda \equiv \frac{r_0}{\pi R^2}$ where R is the radius of the AdS_5 space. From the field theory side the energy scale Λ is related to the expectation value of dimension 4 operator $\mathcal{O} = Tr\Phi_{i_1}\Phi_{i_2}\Phi_{i_3}\Phi_{i_4}$, that is, $\langle \mathcal{O} \rangle \sim \Lambda^4$ which according to the AdS/CFT correspondence can be extracted from the finite part of the conjugate momenta Π [52] through the relation $\langle \mathcal{O} \rangle = \Pi \sim \lim_{r \rightarrow \infty} \sqrt{-g} g^{rr} \partial_r h \sim \Lambda^4$ of the massless metric fluctuation $h = \bar{g}^{\mu\nu} h_{\mu\nu} = 1 - \bar{g}^{\mu\nu} g_{\mu\nu}$, where $\bar{g}_{\mu\nu}$ is the metric component of pure $AdS_5 \times S^5$ space while $g_{\mu\nu}$ is our 10-dimensional metric (29) [19]. Since, we will also normalize $h(r)$ to vanish at the boundary, i.e., $h(r \rightarrow \infty) = 0$, our operator $\langle \mathcal{O} \rangle$ has no physical source at the boundary, rather it gets its VEV dynamically through the Higgs mechanism. Therefore, our new ensemble should be identified as dual to $\mathcal{N} = 4$ SYM on the Coulomb branch (cSYM) at finite-temperature. We will also show that in $\mathcal{N} = 4$ cSYM the energy scale Λ plays similar role as Λ_{QCD} in QCD.

Thermodynamics of $\mathcal{N} = 4$ SYM plasma on the Coulomb branch (cSYM). The action for the $U(1)^3$ consistent truncation of Type IIB supergravity on S^5 is given by [37, 38], see also [39, 40]

$$S = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g_5} \mathcal{L}_{bulk} \quad (1)$$

where

$$\begin{aligned} \mathcal{L}_{bulk} &= (\mathcal{R} - V) - \frac{1}{2} \sum_{I=1}^2 (\partial\varphi_I)^2 - \frac{1}{4} R^2 \sum_{a=1}^3 X_a^{-2} (F^a)^2, \\ F_{\mu\nu}^a &= \partial_\mu A_\nu^a - \partial_\nu A_\mu^a, \quad V = -\frac{4}{R^2} \sum_{a=1}^3 X_a^{-1}, \\ X_1 &= e^{-\frac{1}{\sqrt{6}}\varphi_1 - \frac{1}{\sqrt{2}}\varphi_2}, \quad X_2 = e^{-\frac{1}{\sqrt{6}}\varphi_1 + \frac{1}{\sqrt{2}}\varphi_2}, \quad X_3 = e^{\frac{2}{\sqrt{6}}\varphi_1}. \end{aligned} \quad (2)$$

Note that we have ignored the Chern-Simons term from the action (1) since it does not play any role in our discussion below.

In this Letter, we study in detail the following rotating black 3-brane solution of the above action (1)

$$ds_{(5)}^2 = \frac{r^2}{R^2} H^{1/3} \left(-f dt^2 + dx^2 + dy^2 + dz^2 \right) + \frac{H^{-2/3}}{R^2 f} dr^2, \quad (3)$$

where

$$f = 1 - \frac{r_h^4}{r^4} \frac{H(r_h)}{H(r)}, \quad H = 1 - \frac{r_0^2}{r^2}, \quad (4)$$

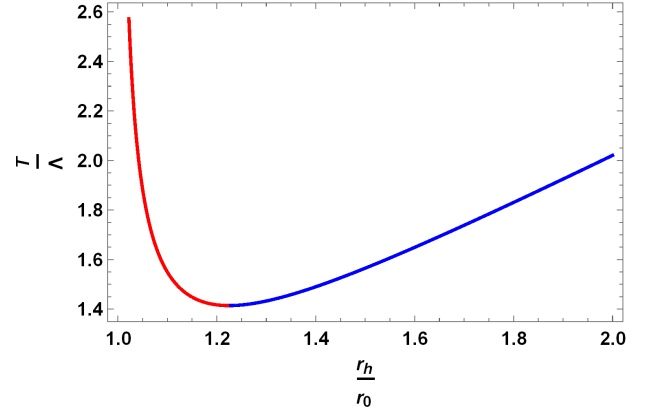


FIG. 1. Hawking temperature $\frac{T}{\Lambda}$ vs. the radius of the horizon $\frac{r_h}{r_0}$ (6), normalized by the energy scale Λ and rotation parameter r_0 , respectively.

$$\begin{aligned} \varphi_1 &= \frac{1}{\sqrt{6}} \ln H, \quad \varphi_2 = \frac{1}{\sqrt{2}} \ln H, \\ A_t^1 &= i \frac{r_0}{R^2} \frac{r_h^2 \sqrt{H(r_h)}}{r^2 H(r)}, \\ r_h^2 &= \frac{1}{2} \left(r_0^2 + \sqrt{r_0^4 + 4m} \right), \end{aligned} \quad (5)$$

$\kappa = \frac{r_0^2}{r_h^2}$, and $A_t^2 = A_t^3 = 0$. Note that our metric (3) is equivalent to the metric used in [33] after analytically continuing $r_0 \rightarrow -i\sqrt{q}$. We should also note that having an imaginary gauge potential, in our ensemble, is not a problem, since all physical quantities in the 5-dimensional spacetime are given in terms of $(\partial_r A_t^1)^2$.

The Hawking temperature T of the black hole (rotating black 3-brane) solution (3) is given by

$$\frac{T}{\Lambda} = \frac{1 - \frac{1}{2}\kappa}{\sqrt{\kappa - \kappa^2}}, \quad (6)$$

where $T_0 = \frac{r_h}{\pi R^2}$, $\Lambda = \frac{r_0}{\pi R^2}$, and $\kappa = \frac{r_0^2}{r_h^2} = \frac{\Lambda^2}{T_0^2}$. We have plotted $\frac{T}{\Lambda}$ in Fig. 1. We can also invert (6) to find

$$\kappa = \frac{1 + \frac{T^2}{\Lambda^2} \left(1 \mp \sqrt{\frac{T^2}{\Lambda^2} - 2} \right)}{\frac{1}{2} + 2 \frac{T^2}{\Lambda^2}}, \quad (7)$$

and

$$\frac{T_0^2}{T^2} = \frac{2 + \frac{1}{2} \frac{\Lambda^2}{T^2}}{1 + \frac{T^2}{\Lambda^2} \left(1 \mp \sqrt{\frac{T^2}{\Lambda^2} - 2} \right)}. \quad (8)$$

Note that in (7) and (8) "−" corresponds to large black hole branch and "+" corresponds to small black hole branch.

The entropy density $s(T, \Lambda)$, for our ensemble where T

and Λ are held fixed, is given by

$$\begin{aligned} s(T, \Lambda) &= \frac{A_H}{4G_5 V_3} = \frac{1}{4G_5} \sqrt{g_{xx}(r_h)g_{yy}(r_h)g_{zz}(r_h)} \\ &= \frac{\pi^2 N_c^2 T_0^3}{2} (1 - \kappa)^{1/2}, \end{aligned} \quad (9)$$

where $G_5 = \pi R^3/2N_c^2$, and V_3 is the three-dimensional volume. And, the corresponding free energy density $f(T, \Lambda)$ of our ensemble can be determined by integrating the entropy density $s(T, \Lambda)$ as [9, 12]

$$f(T, \Lambda) - f(T_{min}, \Lambda) = - \int_{T_{min}}^T s(T', \Lambda) dT', \quad (10)$$

where $T_{min} = \sqrt{2}\Lambda$. Note that it is possible to find the free energy f by simply integrating the entropy density s because the source $h = \bar{g}^{\mu\nu} h_{\mu\nu}$ to $< \mathcal{O} > \sim \Lambda^4$ is normalized to vanish at the boundary.

The integral (10) can be evaluated analytically as

$$\begin{aligned} f(T, \Lambda) &= - \int_{r_{hmin}}^{r_h} \frac{dT'}{dr'_h} s(r'_h, \Lambda) dr'_h \\ &= - \frac{\pi^2 N_c^2 T_0^4}{8} \left(1 - \kappa - \frac{3}{4} \kappa^2 - \kappa^2 \log\left(\frac{2}{\kappa} - 2\right) \right) \end{aligned} \quad (11)$$

where we choose $r_{hmin} = \sqrt{\frac{3}{2}} r_0$, and set the integration constant $f(T_{min}, \Lambda) = 0$. We have plotted the free energy density $f(T, \Lambda)$ (10) in Fig. 2

The other thermodynamic quantities can be determined from the free energy density $f(T, \Lambda)$ (11) as: pressure $p = -f$, energy density $\epsilon = p + Ts$, specific heat $C_\Lambda = T \left(\frac{\partial s}{\partial T} \right)_\Lambda$, and speed of sound $c_s^2 = \frac{\partial p}{\partial \epsilon} = \frac{s}{C_\Lambda}$. We have plotted the thermodynamics quantities in Fig. 4, Fig. 5. To compare our results with pure Yang-Mills theory on the lattice and improved holographic QCD see Fig.5-9 in [12].

Holographic hadronization, thermal hadron emission rate and black hole information paradox. The EoS plotted in Fig. 4 has interesting physical interpretation, that is, a localized large black hole expands and cools down due to its internal pressure $p(T)$ until its temperature reaches the critical temperature T_c at which $p(T_c) = 0$, and the large black hole smoothly turns in to a localized small black hole (a second-order phase transition). Since, the small black hole has negative specific heat and is unstable even classically, it starts Hawking radiating hadrons and eventually its energy and entropy vanishes and turn in to thermal AdS or gas of hadrons. Since, this entire process (the production of plasma in a hadron-hadron collision, and its subsequent decay via hadronization) is unitary, there is no information loss. Therefore, the small black hole information paradox [41, 42] is resolved in AdS/CFT correspondence.

As a comparison to $\mathcal{N} = 4$ cSYM, let us look at what happens for $\mathcal{N} = 4$ SYM on sphere based on its free

energy density f_{sphere} plotted in Fig. 3 and given by [16], see also [45],

$$f_{sphere} = \frac{F_{sphere}}{V_3} = - \frac{\pi^2 N_c^2 T_0^4}{8} (1 - \kappa_{sphere}), \quad (12)$$

where $\kappa_{sphere} = \frac{R^2}{r_h^2} = \frac{\Lambda_{sphere}^2}{T_0^2}$, and the Hawking temperature $\frac{T}{\Lambda_{sphere}} = \frac{1 + \frac{1}{2} \kappa_{sphere}}{\sqrt{\kappa_{sphere}}}$. A localized large black hole with spherical horizon expands and cools down due to its internal pressure $p_{sphere}(T) = -f_{sphere}$ until its temperature reaches the critical temperature T_c at which $p_{sphere}(T_c) = 0$ and abruptly changes to thermal AdS (a first-order phase transition also known as Hawking-Page transition [16, 46]). Since, this entire process is unitary and large black holes are classically stable and will not evaporate away, there is no information loss in $\mathcal{N} = 4$ SYM on sphere and its holographic dual, see [47, 48].

We have seen that $\mathcal{N} = 4$ cSYM plasma in its hadronizing phase behave like Schwarzschild black hole in asymptotically flat 4-dimensional spacetime. This observation motivates us to make a second conjecture that the small black hole in asymptotically AdS space is a holographic dual to Schwarzschild black hole in asymptotically flat 4-dimensional spacetime which is equivalent to $\mathcal{N} = 4$ cSYM plasma in its hadronizing phase.

Hence, according to our second conjecture, Hawking radiation from the Schwarzschild black hole in 4D is nothing but thermal hadron emission, hence there is no information loss even in the Schwarzschild black hole in 4D, at least when the 4D Universe is governed by $\mathcal{N} = 4$ cSYM theory.

A strong evidence to our second conjecture can be found by looking at the g_{tt} component of the 10-dimensional metric (29) at $\theta = \frac{\pi}{2}$ and near the horizon $r \rightarrow r_h$ limit

$$g_{tt} = -8 \frac{r^2}{R^2} \left(\left(1 - \frac{r_h}{r} \right)^2 + \frac{r_h^2}{8r^2} + \mathcal{O}((r - r_h)^3) \right), \quad (13)$$

which can be expanded near the boundary $r \rightarrow \infty$ as

$$g_{tt} = -8 \frac{r^2}{R^2} \left(1 - \frac{2r_h}{r} + \mathcal{O}(1/r^2) \right), \quad (14)$$

where we subtracted off the UV divergent terms. Similarly, one can show that $g_{rr} = \frac{R^2}{8r^2} \left(1 - \frac{2r_h}{r} + \mathcal{O}(1/r^2) \right)^{-1}$. Note that the heavy quark-antiquark potential $V(L)$, in this 10-dimensional background metric (29), is confining only for $\theta = \frac{\pi}{2}$ [20], which we will discuss in more detail in the next sections.

Therefore, near the horizon, the 5D part of (29) is given by

$$ds^2 = -8 \frac{r^2}{R^2} \left(1 - \frac{2r_h}{r} \right) dt^2 + \frac{1}{8 \frac{r^2}{R^2} \left(1 - \frac{2r_h}{r} \right)} dr^2 + \frac{r_h^2}{R^2} d\vec{x}^2. \quad (15)$$

The near horizon metric (29) describes the small black hole in AdS which is conjectured to be the holographic dual to the Schwarzschild black hole in 4D with Newtonian gravitational potential $\Phi(r) \equiv -\frac{r_h}{r} = -\frac{G_4 M}{r}$ where G_4 is Newton's gravitational constant in 4D and M is the mass of the Schwarzschild black hole or $\mathcal{N} = 4$ cSYM plasma in its hadronizing phase.

Hence, a hadron of mass M_h near the hadronizing $\mathcal{N} = 4$ cSYM plasma of mass M or 4-dimensional Schwarzschild black hole of mass M experiences an emergent attractive force which we call Newton's gravitational force given by

$$F = \Phi'(r)M_h = G_4 \frac{MM_h}{r^2}. \quad (16)$$

Our second conjecture makes precise the previous proposal by Nastase that RHIC fireball is the analog of Schwarzschild black hole in asymptotically flat 4-dimensional spacetime [43, 44].

Finally, we propose the following formula to compute the number of thermal hadrons emitted per unit time per unit volume in the local rest frame from $\mathcal{N} = 4$ cSYM plasma in its hadronizing phase

$$\frac{d\Gamma_{hadron}}{d^3k} \equiv \frac{1}{\sigma(0)} \frac{d\Gamma_{Hawking}}{d^3k}, \quad (17)$$

where [49]

$$(2\pi)^3 \frac{d\Gamma_{Hawking}}{d^3k} = \frac{\sigma(\omega)}{e^{\omega/T} \mp 1}, \quad (18)$$

is the Hawking radiation rate of the small black hole, and $\sigma(\omega)$ is the cross-section for a hadron (bulk fluctuation) of energy ω coming in from infinity to be absorbed by $\mathcal{N} = 4$ cSYM plasma in its hadronizing phase (small black hole). The absorption cross-section $\sigma(\omega)$ is given by

$$\sigma(\omega) = -\frac{16\pi G_5}{\omega} \text{Im} G^R(\omega), \quad (19)$$

where $G^R(\omega)$ is the retarded two-point function of an operator \mathcal{O} corresponding to the hadron, [50]

$$G^R(\omega) = -i \int dt d\mathbf{x} e^{-i\omega t} \theta(t) \langle [\mathcal{O}(t, \mathbf{x}), \mathcal{O}(0, 0)] \rangle, \quad (20)$$

which can be computed using the dictionary of AdS/CFT correspondence [51, 52]. For example, for spin-2 glueballs $\mathcal{O} = T_x^y$ is the energy-momentum tensor operator with a source transverse metric bulk fluctuation (graviton) h_y^x .

In the hydrodynamic limit $\omega \ll T$, (17) reduces to the Cooper-Frye formula in the local rest frame

$$(2\pi)^3 \frac{d\Gamma_{hadron}}{d^3k} \simeq \frac{1}{e^{\omega/T} \mp 1}, \quad (21)$$

which is nothing but a rest frame Bose or Fermi distribution, see Eq.A1 of [53]. Note that $\omega^2 = M_h^2 + |\mathbf{k}|^2$ where M_h is the mass of the hadron and \mathbf{k} is its momentum.

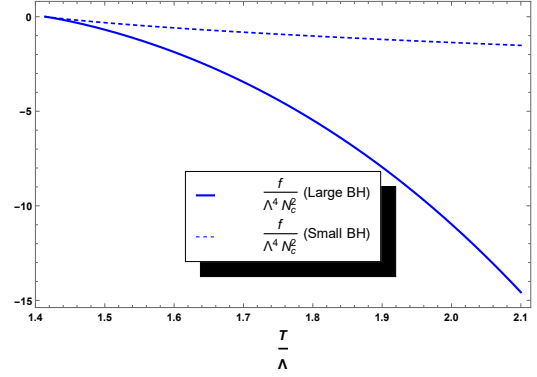


FIG. 2. The free energy density $\frac{f}{\Lambda^4 N_c^2}$ of $\mathcal{N} = 4$ SYM plasma on the Coulomb branch (11), normalized by $\Lambda^4 \sim \langle \mathcal{O} \rangle$, and N_c^2 , for both large and small black holes.

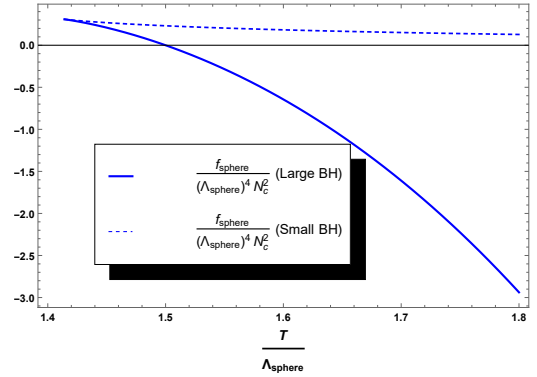


FIG. 3. The free energy density $\frac{f_{sphere}}{(\Lambda_{sphere})^4 N_c^2}$ of $\mathcal{N} = 4$ SYM plasma on 3-sphere of radius R (12), normalized by $(\Lambda_{sphere})^4 = \frac{1}{\pi^4 R^4}$, and N_c^2 , for both large and small black holes.

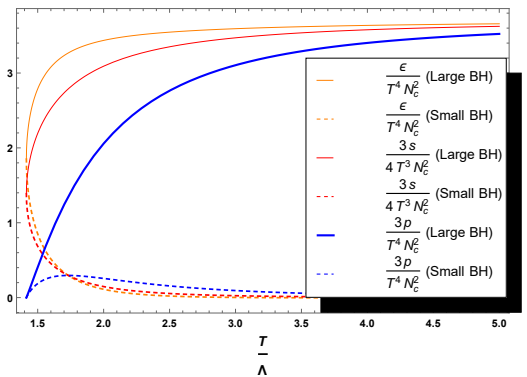


FIG. 4. The energy density $\frac{\epsilon}{T^4}$, entropy density $\frac{3}{4} \frac{s}{T^3 N_c^2}$, and pressure $\frac{3}{4} \frac{p}{T^4 N_c^2}$ of $\mathcal{N} = 4$ SYM plasma on the Coulomb branch for both large and small black hole branches. Note that in the conformal or high temperature limit $\frac{\epsilon}{T^4} = \frac{3}{4} \frac{s}{T^3 N_c^2} = \frac{3}{4} \frac{p}{T^4 N_c^2}$ for both large and small black holes.

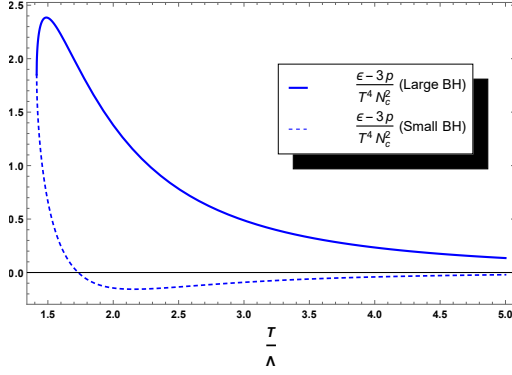


FIG. 5. The trace anomaly $\frac{\epsilon-3p}{T^4 N_c^2}$ of $\mathcal{N} = 4$ SYM plasma on the Coulomb branch for both large and small black holes.

Cornell potential in $\mathcal{N} = 4$ cSYM. The Nambu-Goto (NG) action is

$$S_{NG} = \int d\tau d\sigma \mathcal{L}(h_{ab}) = -\frac{1}{2\pi\alpha'} \int d\tau d\sigma \sqrt{-\det h_{ab}}, \quad (22)$$

where the background induced metric on the string h_{ab} is given by

$$h_{ab} = g_{\mu\nu} \partial_a x^\mu(\tau, \sigma) \partial_b x^\nu(\tau, \sigma). \quad (23)$$

Using the embedding $(\tau, \sigma) \Rightarrow (t(\tau, \sigma), 0, 0, x(\tau, \sigma), r = \sigma)$, the background induced metric $h_{ab}(x')$ (23) becomes ($' \equiv d/d\sigma$)

$$\begin{aligned} h_{\tau\tau}(x') &= g_{tt}, \\ h_{\sigma\sigma}(x') &= g_{rr} \left(\frac{1}{1 + \frac{C^2}{g_{xx}g_{tt}}} \right), \end{aligned} \quad (24)$$

where we used

$$(x')^2 = \frac{-C^2 g_{rr}}{g_{xx}^2 g_{tt}} \frac{1}{\left(1 + \frac{C^2}{g_{tt}g_{xx}}\right)}. \quad (25)$$

which is the solution of the NG equation of motion, and the integration constant C is related to the conjugate momenta $\Pi = \frac{\partial \mathcal{L}}{\partial x'} = -\frac{C}{2\pi\alpha'}$.

Considering a string configuration where a heavy quark is attached to each ends of the string, we can extract the potential energy $V(L)$, of the two quarks separated by length L , from the on-shell Nambu-Goto action S_{NG} as

$$V(L) = \frac{-2S_{NG}}{\mathcal{T}}, \quad (26)$$

where

$$\begin{aligned} -\frac{2\pi\alpha'}{\mathcal{T}} S_{NG} &= \int_{r_m}^{\infty} dr \sqrt{-\det h_{ab}(x')} - \int_0^{\infty} dr \sqrt{-\det h_{ab}(0)} \\ &= \int_{r_m}^{\infty} dr \left(\sqrt{-\det h_{ab}(x')} - \sqrt{-\det h_{ab}(0)} \right) \\ &\quad - \int_{r_h}^{r_m} dr \sqrt{-\det h_{ab}(0)}, \end{aligned} \quad (27)$$

and r_m is related to L through the boundary condition $\frac{L}{2} = \int_{r_m}^{\infty} x' dr$, and we also fix the integration constant C by demanding $x'|_{r=r_m} \rightarrow \infty$ which is satisfied only when $C^2 = -g_{tt}(r_m)g_{xx}(r_m)$. Note that we have a factor of 2 in (7) because our gauge covers only half of the full string configuration which accounts to only half of the full potential energy between the quarks, see [45] for discussion on how to compute $V(L)$ in the $x(r)$ gauge instead of the widely used $r(x)$ gauge of [54].

For $r \gg r_m$ and in the extremal limit where $r_h = r_0$ or $f = 1$, we can approximate $h_{\sigma\sigma}(x') \cong h_{\sigma\sigma}(0) = g_{rr}$, $x' \cong \frac{g_{xx}(r_m)}{g_{xx}} \sqrt{\frac{g_{rr}}{g_{xx}}} \cong \frac{r_m^2 R^2}{r^4}$, and

$$\begin{aligned} V(L) &\simeq -\frac{1}{\pi\alpha'} \int_{r_0}^{r_m} dr \sqrt{-\det h_{ab}(0)} = -\frac{1}{\pi\alpha'} \int_{r_0}^{r_m} dr H^{-\frac{1}{6}} \\ &\simeq -\frac{r_m}{\pi\alpha'} + \frac{1}{6\pi\alpha'} \frac{r_0^2}{r_m} + \frac{5r_0}{6\pi\alpha'} + \mathcal{O}(r_0^4) \\ &\simeq -\frac{2\sqrt{\lambda}}{3\pi} \frac{1}{L} + \frac{\pi\sqrt{\lambda}\Lambda^2}{4} L + \frac{5\Lambda}{6} + \mathcal{O}(r_0^4), \end{aligned} \quad (28)$$

where we used $\frac{L}{2} = \int_{r_m}^{\infty} x' dr \cong \frac{1}{3} \frac{R^2}{r_m}$ to get the last line.

Uplifting the 5D metric to 10D. The 5-dimensional metric (3) can be uplifted to the full 10-dimensional metric as [19, 20, 32]

$$\begin{aligned} ds_{(10)}^2 &= \frac{r^2}{R^2} \tilde{H}^{1/2} \left(-\tilde{f} dt^2 + dx^2 + dy^2 + dz^2 \right) + \frac{\tilde{H}^{1/2} H^{-1}}{R^2 \tilde{f}} dr^2 \\ &\quad + R^2 \left(\tilde{H}^{1/2} d\theta^2 + H \tilde{H}^{-1/2} \sin^2 \theta d\phi^2 + \tilde{H}^{-1/2} \cos^2 \theta d\Omega_3^2 \right) \\ &\quad + 2A_t \tilde{H} \tilde{H}^{-1/2} R^2 \sin^2 \theta dt d\phi, \end{aligned} \quad (29)$$

where

$$\tilde{H} = \sin^2 \theta + H \cos^2 \theta, \text{ and } \tilde{f} = 1 - \frac{r_h^4}{r^4} \frac{H(r_h)}{\tilde{H}(r)}, \quad (30)$$

f and H are the same as in (3). Our 10-dimensional metric (29) is equivalent to Eq.2.21 of [32] after analytically continuing the rotation parameter $r_0 \rightarrow -ir_0$, and re-writing (29) in terms of $\mu \equiv m^{1/4}$. Note that the $g_{t\phi}$ component of (29) is imaginary and one could make it real by analytically continuing $t \rightarrow -it$ as in [20, 29]. However, since we are also interested in real-time dynamics, such as computation of transport coefficients [58], we refrain from analytically continuing $t \rightarrow -it$, and we treat our 10-dimensional metric (29) as a complex saddle point. Also note that $g_{t\phi} = A_t^1 = 0$ in the extremal limit $r_h = r_0$.

In [20], the heavy quark-antiquark potential energy $V(L)$ was computed for the 10-dimensional background metric (29) after analytically continuing $t \rightarrow -it$ and in the extremal limit where $r_h = r_0$ or $\tilde{f} = f = 1$ case. The authors have shown that, for $\theta = \frac{\pi}{2}$, $V(L)$ smoothly interpolates between a Coulombic potential $V(L) = -\frac{2\Gamma(3/4)^2 \sqrt{\lambda}}{\Gamma(1/4)^2} \frac{1}{L}$ for small L and a confining potential $V(L) = \frac{\pi\sqrt{\lambda}\Lambda^2}{2} L$ for large L . See curve (b) in

Fig.5 of [20]. Their numerical result agrees qualitatively with our analytic result (28) on the 5-dimensional metric (29).

Glueballs in $\mathcal{N} = 4$ cSYM. It can easily be shown that bulk fluctuations in the 5-dimensional metric (3), at least in the near boundary limit where the metric is essentially AdS_5 space with IR cut-off at $r = r_0$, have mass-gap and quantized mass spectrum proportional to $\Lambda = \frac{r_0}{\pi R^2}$.

In [20], it was shown that a scalar bulk fluctuation in a 10-dimensional metric (29), after analytically continuing $t \rightarrow -it$, indeed has mass gap proportional to Λ and a quantized mass spectrum $M_n^2 = 4\pi^2\Lambda^2n(n+1)$, see Eq.54 in [20]. Since, a scalar bulk fluctuation in (29) has the same 5-dimensional bulk equation of motion as in [20] which is the Jacobi equation, we can use this result to calculate the mass spectrum of glueballs in $\mathcal{N} = 4$ cSYM.

The transverse gravitational tensor fluctuation $h_y^x(t, z, r)$ in the 10-dimensional metric (29), which is a source to dimension 4 stress-energy tensor operator T_x^y , also has the same 5-dimensional bulk equation of motion as the scalar field which is the Jacobi equation. Therefore, we can infer that the operator T_x^y which corresponds to spin-2 glueballs of $J^{PC} = 2^{++}$ [55] has mass spectrum given by $M_n^2 = 4\pi^2\Lambda^2n(n+1)$ for $n = 1, 2, \dots$

The real and imaginary parts of the bulk fluctuation of a massless complex scalar field $\Phi = e^{-\phi} + i\chi$, in the 10-dimensional metric (29), are sources to the dimension 4 scalar operators $\mathcal{O}_4 = Tr F^2$ and $\tilde{\mathcal{O}}_4 = Tr F \wedge F$, respectively [56], and its 5-dimensional bulk equation of motion is the Jacobi equation. Therefore, \mathcal{O}_4 and $\tilde{\mathcal{O}}_4$ which correspond to the scalar glueballs of $J^{PC} = 0^{++}$ and $J^{PC} = 0^{-+}$, respectively, have a degenerate mass spectrum given by $M_n^2 = 4\pi^2\Lambda^2n(n+1)$ for $n = 1, 2, \dots$

Summary and discussion. We have shown that the large black hole branch of the non-extremal rotating black 3-brane background solution (3) has pure Yang-Mills-like equation of state: the pressure p vanishes at critical temperature $T_c = T_{min} = \sqrt{2}\Lambda$, see Fig. 4; the trace anomaly $\epsilon - 3p$ have a maxima around T_c and vanishes at very high temperature, see Fig. 5; and the speed of sound c_s^2 approaches its conformal limit $1/3$ from below. In order to compare our results with pure Yang-Mills theory on the lattice and improved holographic QCD see Fig.5-9 in [12].

Moreover, we have shown that there is a second-order phase transition from the large black hole branch, of the rotating black 3-branes, with radius $r_h > \sqrt{3}/2r_0$ and positive specific heat capacity $C_\Lambda > 0$ to the small black hole branch with radius $r_h < \sqrt{3}/2r_0$ and negative specific heat capacity $C_\Lambda < 0$. Since, the small black hole has negative specific heat capacity, it is unstable and decays by emitting Hawking radiation. It can be seen in Fig. 4 that the entropy density, energy density, and pressure of the small black hole decreases due to the Hawking

radiation.

From the gravity side, the reason why we have this second-order phase transition is due to the fact that, as can be seen from the free energy depicted in Fig. 2, the small black hole branch has lower (or negative) free energy compared to the thermal AdS space or extremal black hole (which can be determined from the black hole solution by setting the mass parameter $m = 0$ or $r_h = r_0$ in (3) and has zero thermal free energy).

From the field theory side, we conjecture that the small black hole should be identified as dual to a hadronizing $\mathcal{N} = 4$ cSYM plasma where the colored scalar, quark, and gluon degrees of freedom of the plasma are turning into colorless hadrons, see [57] for analogous phenomena where plasma-balls decay by thermally radiating hadrons. The hadronization process (Hawking radiation) decreases the color degrees of freedom of the plasma resulting in the decrease of the entropy and energy densities of the plasma, see Fig. 4. We have proposed a formula (17) which relates the Hawking radiation rate with the thermal hadron emission rate, and in the hydrodynamic limit $\omega \ll T$ reduces to Cooper-Frye formula in the local rest frame.

Since $\mathcal{N} = 4$ cSYM plasma in its hadronizing phase behave like Schwarzschild black hole in asymptotically flat 4-dimensional spacetime, we have made a second conjecture that the small black hole in asymptotically AdS space is a holographic dual to Schwarzschild black hole in 4D, and argued that there is no information loss even in the Schwarzschild black hole in 4D since its Hawking radiation is nothing but thermal hadron emission, at least in a 4D Universe governed by $\mathcal{N} = 4$ cSYM.

We have provided strong evidence to the second conjecture by deriving the Newton's gravitational force a hadron of mass M_h experiences near $\mathcal{N} = 4$ cSYM plasma of mass M , in its hadronizing phase (16).

We have computed the heavy quark-antiquark potential energy $V(L)$ of $\mathcal{N} = 4$ cSYM at $T = 0$ and have shown that it is given by Cornell potential, see (28). We have also shown that the mass spectrums of the scalar and spin-2 glueballs of $\mathcal{N} = 4$ cSYM at $T = 0$ are degenerate and given by $M_n^2 = 4\pi^2\Lambda^2n(n+1)$ for $n = 1, 2, \dots$

Note that we have investigated the hydrodynamic transport coefficients and hard probes of the strongly coupled $\mathcal{N} = 4$ cSYM plasma in [58].

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